Self-referencing languages revisited

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Outline

1. Self-referencing as it conventionally formalized
2. Why is it wrong?
3. Self-referencing rephrased in iCTRL
4. Conclusions

Tarski's paradox (traditional language of logic)

Truth definition

$$t(\ulcorner x \urcorner) \equiv x$$

Liar sentence representation alternatives

$$l \equiv \sim t(\ulcorner l \urcorner)$$
$$\ulcorner l \urcorner = \ulcorner \sim t(\ulcorner l \urcorner) \urcorner$$

Contradiction

$$t(\ulcorner l \urcorner) \equiv \sim t(\ulcorner l \urcorner)$$
$$l \equiv t(\ulcorner l \urcorner) \equiv t(\ulcorner \sim t(\ulcorner l \urcorner) \urcorner) \equiv \sim t(\ulcorner l \urcorner)$$

Defects of liar defintions

According to propositional logic

$$x \equiv t(\ulcorner x \urcorner) \vdash \sim (x \equiv \sim t(\ulcorner x \urcorner))$$

Then the formula defining the liar sentence is *false*

$$\sim (l \equiv \sim t(\ulcorner l \urcorner))$$

This result does not change if $\equiv$ is replaced by $=$, because from Leibniz's rule

$$(\ulcorner x \urcorner = \ulcorner y \urcorner) \supset (x \equiv y)$$
$$\sim (x \equiv \sim t(\ulcorner x \urcorner)) \supset \sim (\ulcorner x \urcorner = \ulcorner \sim t(\ulcorner x \urcorner) \urcorner)$$

Which also yields that the liar sentence is *false*

$$\sim (\ulcorner l \urcorner = \ulcorner \sim t(\ulcorner l \urcorner) \urcorner)$$

The problem

The formula defining the liar sentence is *false*, it is an inconsistent identifying description for the liar sentence (fallacy of definition).

Moreover, **there is no proposition** l, which can satisfy either $x \equiv \sim t(\ulcorner x \urcorner)$ or $\ulcorner x \urcorner = \ulcorner \sim t(\ulcorner x \urcorner) \urcorner$.

Truth definition itself seems to exclude the problematic proposition, which was constantly the goal of various solution routes trying to expel it from formal language.

Still, natural language can really express the liar sentence, and the paradox really arises.

Does any adequate representation of the liar sentence exist?

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What is wrong?

The original liar sentence is inconsistent as a definition

(1) is not true. (1) $l \equiv \sim t(\ulcorner l \urcorner)$

$\ulcorner l \urcorner = \ulcorner \sim t(\ulcorner l \urcorner) \urcorner$

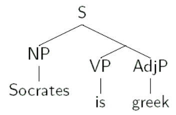
Antinomy is still present in natural languages.

The representation language may be defective.

- Traditional language of logic is a truth value invariant mapping.
- Syntax is mapped diffusely.

Sentence vs. formula

Socrates is greek.	$greek(Socrates) \equiv \exists x(x = Socrates \,\&\, greek(x))$
Human is mortal.	$\forall x.human(x) \supset mortal(x)$
Some human is white.	$\exists x.human(x) \,\&\, white(x)$
Two and two are four.	$2 + 2 = 4$



- The predicate-subject relation is subordination.
- Logical connectives represent co-ordination.
- Traditionally, subordination is also represented by co-ordination.
- Loss of information may arise, readability is damaged.

Change representation language

Targets

- Semantical invariance: the **truth value invariant** mapping should be preserved.
- Syntactical invariance: a **natural language relation invariant** mapping is preferred.

Possible advantages

- A higher level fidelity of translation.
- Loss of information may be eliminated.

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Self-referencing rephrased in iCTRL

α is true. $t\ x, \langle\alpha\rangle x.$

- $t\ x$ denotes the truth predicate
- α stands for an arbitrary proposition
- $\langle\alpha\rangle x$ represents the name of α
- $t\ x, \langle\alpha\rangle x.$ is a closed formula
- $|t\ x, \langle\alpha\rangle x.| = true \equiv r(\langle\alpha\rangle x) \subseteq r(t\ x)$
- Only the language of predicate logic has been changed!

Truth definition:

$$t\ x, \langle\alpha\rangle x. \equiv \alpha$$

Ordinary reference

Referring to subjects

$$John\ walks.$$
$$He\ whistles.$$

He refers to the subject of the previous sentence, i.e. *John*

$$walk\ x, John\ x.$$
$$whistle\ y : x.$$

The reference variable can easily be resolved by substitution: find the formula subject having the same variable symbol as the one to right of the colon, and insert it replacing its variable symbol by the one to the left of the colon.

Self-reference

Liar sentence, representing self-reference:

(1) is not true. (1) $\langle \sim t\ x : y \rangle y.$

- $\sim t\ x : y$ is the predicate of the formula.
- The subject that is referred to is the formula itself: $\langle \sim t\ x : y \rangle y.$

Resolving reference yields:

$$\sim t\ y, \langle \sim t\ x : y \rangle y.$$

Getting liar paradox

Let $\langle \sim t\ x : y \rangle y.$ be evaluated *true*

Then resolving reference yields $\sim t\ y, \langle \sim t\ x : y \rangle y.$

Substituting this to the truth definition, $t\ x, \langle \alpha \rangle x. \equiv \alpha$, the result is $\sim \langle \sim t\ x : y \rangle y.$

Let $\sim \langle \sim t\ x : y \rangle y.$ be evaluated *true*

Then resolving reference yields $t\ y, \langle \sim t\ x : y \rangle y.$

Substituting this to the truth definition, $t\ x, \langle \alpha \rangle x. \equiv \alpha$, the result is $\langle \sim t\ x : y \rangle y.$

Consequently, we have proven

$$\langle \sim t\ x : y \rangle y \equiv \sim \langle \sim t\ x : y \rangle y$$

Summary

Liar sentence translated by iCTRL avoids the fault of inconsistent, contradictory defining.

It is a clear, faithful, word-for-word representation.

It can identify antinomy relative to truth definition.

The question of solution remains open, needs further research.

Outline

Conclusions

Traditional language of logic fails to represent the liar sentence.

This is not a singular case when traditional language of logic fails to represent a phenomenon correctly (–: Ordinary and Intensional CTR, in Zreik, K. ed.: Intn'l Workshop on Phil. of Design & IT, Château du Baffy, (1994), 109-122. –: iCTRL ..., *AI*, **109**/1-2 (1999), 33-70.).

- Handling double necessity in S4 axiom: $Np \supset NNp$
- Analysing validity conditions of the converse of Barcan formula: $\exists x.M[\alpha(x)] \supset M[\exists x.\alpha(x)]$

iCTRL can adequately represent this sentence and the paradox.

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Liar sentence and languages

	Liar sentence	Paradox
Natural language	$\exists$	$\exists$
Non-restrictive formal language	$\exists \rightarrow ?$	$\exists \rightarrow ?$
Restrictive formal language	$\sim\exists$	$\sim\exists$